Efficient Feedback Design for MIMO SC-FDMA Systems

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Abstract

An efficient feedback design for single carrier frequency division multiple access (SC-FDMA) multiple input multiple output (MIMO) communication systems is described. We consider a codebook-based approach for quantization of feedback information. Differential transform processing is used for feedback design to reduce feedback overhead. Jacobi rotation is utilized for generating the MIMO pre-coding matrix and feedback information. Differential processing is used to generate the updates iteratively for the feedback information between feedback intervals. An approach of combined differential and non-differential feedback is considered for reduced feedback overhead and improved performance. A differential feedback with periodic error reset for differential processing is described. The performance of MIMO communication systems using pre-coding/transmit beamforming and the described feedback design is evaluated for SC-FDMA systems. The effects of pre-coding and feedback quantization, sub-carrier grouped feedback and feedback delay are investigated.

Index Terms— **MIMO, feedback, pre-coding, beamforming, SC-FDMA.**

1. Introduction

MIMO precoding is an efficient scheme for providing high data rate and increased system capacity [1], [2]. It provides better performance than MIMO techniques using spatial multiplexing without precoding or beamforming [3]. However the actual performance of MIMO precoding depends on the availability of accurate and timely the channel state information (CSI) at the transmitter. In order to perform precoding or transmit beamforming (TxBF) the transmitter needs to know either the CSI for MIMO channels or the precoding matrix. In a closed-loop MIMO precoding system the receiver estimates the CSI via a training sequence or pilot which is known to receiver.

The precoding or beamforming matrix can be obtained from CSI for MIMO channels using singular value decomposition (SVD) [4], [5]. In a time division duplex (TDD) system the estimated CSI or the precoding matrix can be obtained using sounding or similar techniques because of the reciprocal channels in both uplink (UL) and downlink (DL). For frequency division duplex (FDD) systems closed-loop feedback is required for the transmitter to obtain such information. The receiver can feedback either the estimated CSI or the precoding matrix that is derived from estimated CSI. Quantization for estimated CSI or precoding matrix is required.

An obvious method to quantize would be to use sufficient bits to represent each element of the estimated CSI or precoding matrix. However such an approach requires a large number of bits for quantization and significantly increased feedback overhead. A feedback technique for MIMO precoding that provides low overhead and high performance is desirable. Some feedback schemes consider statistical feedback for reduced feedback overhead such as mean and covariance feedback in which channel spatial covariance can be obtained without overhead by averaging the other link measurement and channel realization for subcarriers require explicit feedback [7]. Some finite rate feedback schemes are also investigated in [8]. Quantization using a codebook approach is discussed in [9].

In this paper an efficient feedback scheme using differential transform processing is analyzed. The MIMO channels are estimated via pilots. The estimated channels are transformed into a precoding matrix using Jacobi rotation. In the proposed feedback scheme, feedback of precoding matrix is equivalent to feedback of those parameters for Jacobi rotation for MIMO precoding. The feedback of parameters for Jacobi rotation is more efficient than feedback of the entire precoding matrix, or the precoding vectors themselves. To further reduce feedback overhead differential processing is introduced in which only the changes of the precoding matrix between updates are computed, and fed back. Differential feedback is efficient for tracking the change of MIMO channels. Differential processing is derived using iterative Jacobi rotation. To avoid error accumulation and propagation introduced by differential processing, an approach that combines differential and non-differential feedback is considered in which a differential feedback with periodic error reset is proposed.

The described techniques can be applied to both uplink and downlink wireless communications for MIMO configuration of two transmit and two receive antennas. With appropriate modifications the described feedback scheme can be applied for MIMO configuration having more antennas.

The paper is organized as follows. Section 2 describes the feedback scheme using differential feedback and Jacobi rotation. Section 3 provides numerical results that present the performance of this feedback scheme for MIMO precoding. The performance degradation due to feedback quantization, sub-carrier group feedback and feedback delay are studied and quantified. The combined effects of quantization, group feedback and feedback delay for MIMO precoding using the described feedback technique is compared with MIMO precoding using ideal feedback assuming no quantization or infinite precision for quantization and no feedback delay. Conclusions are summarized in Section 4.

2. Feedback scheme description

The MIMO channels in frequency domain for two transmit and two receive antennas in a SC-FDMA system [11][12] can be expressed by a matrix,

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \qquad (1)$$

where h_{ij} represents the channel response between j^th^ antenna at transmitter and ith antenna at receiver. MIMO channels H can be estimated via pilot or training sequences. When grouped feedback is used, the channels are averaged across subcarriers. Suppose one feedback per L subcarriers is used. The channel responses are first averaged for L subcarriers. Denote the resulting averaged channel response estimates as $\hat{H}$. The averaged estimated channel response matrix $\hat{H}$ can be decomposed using SVD by

$$\hat{H} = UDV^H \qquad (2)$$

where U and V are the unitary matrix, i.e., $U^H U = I$ and $V^H V = I$. D is a diagonal matrix that has singular values in the diagonal. The channel correlation matrix is obtained by multiplying the Hermitian transpose of the estimated channel response matrix $\hat{H}$ with $\hat{H}$ itself as

$$R = \hat{H}^H \hat{H} \qquad (3)$$

Substituting the estimated channel response matrix $\hat{H}$ in equation 3, the channel correlation matrix R can be rewritten as

$$R = VD^2 V^H \qquad (4)$$

When comparing equations 3 and 4, it can be seen that diagonalizing the channel response matrix $\hat{H}$ to find the eigen-matrix V is equivalent to diagonalizing the channel correlation matrix R.

Alternatively Jacobi rotation is used to perform the matrix diagonalization of channel correlation matrix R such that

$$J^H RJ = D^2 \qquad (5)$$

It is observed that the Jacobi rotation matrix J can be used as a precoding matrix when comparing equations 4 and 5 if equation 5 is multiplied by a Jacobi rotation matrix J from the left side and a Hermitian transpose of Jacobi rotation matrix from the right size on both left and right terms of the equation. The Jacobi rotation or transform matrix for 2 x 2 MIMO configuration is represented as

$$J(\theta,\phi) = \begin{bmatrix} \cos(\theta)e^{j\phi} & \sin(\theta)e^{j\phi} \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \qquad (6)$$

where θ and ϕ are parameters for Jacobi rotation. The parameters θ and ϕ can be obtained by the equations

$$\tan(\theta)^2 + \frac{(r_{22} - r_{11})}{|r_{12}|}\tan(\theta) - 1 = 0 \qquad (7)$$

$$e^{j\varphi} = \frac{r_{12}}{|r_{12}|} \qquad (8)$$

where r_{ij} is the element of channel correlation matrix R that corresponds to the i^th^ row and j^th^ column.

To further reduce feedback overhead a differential feedback using iterative Jacobi transform is described. For feedback instance *n*, the Jacobi rotation *J(n)* is applied on channel correlation matrix R and is expressed by

$$J(n)^H R(n) J(n) = D^2 \qquad (9)$$

For the next feedback instance *n+1*, if the Jacobi rotation matrix is not updated, diagonalization of matrix *R* using Jacobi rotation of feedback instance n can be expressed by

$$J(n)^H R(n+1) J(n) = \tilde{D}^2 \qquad (10)$$

When MIMO channels change, $\tilde{D}^2$ is not diagonal. The precoding matrix and, therefore, Jacobi rotation matrix needs to be updated for correct diagonalization. Call *J* the differential precoding matrix that represents the delta of the feedback update whose parameters θ and ϕ are sent back to the transmitter from the receiver. The previous precoding matrix *J(n)* at transmitter is updated to obtain the next precoding matrix *J(n+1)* by multiplying the previous precoding matrix *J(n)* with the differential precoding matrix J,

$$J(n+1) = J(n)J \qquad (11)$$

The differential feedback *J* can be computed at the receiver by multiplying the Hermitian transpose of the previous precoding matrix *J(n)* with the current precoding matrix *J(n+1*) by

$$J = J(n)^H J(n+1) \qquad (12)$$

where *J(n+1)* can be computed from correlation matrix *R(n+1)* at feedback instance *n+1*.

In general differential feedback may be more suitable for low speed channels and non-differential feedback may be more suitable for high speed channels. A combined differential and non-differential feedback may be considered for feedback overhead reduction and performance improvement.

Note that differential feedback can be reset after a certain period of time, e.g. every N transmission time intervals (TTIs), for avoiding error accumulation or propagation due to differential processing and feedback bit errors/erasures. At each reset non-differential feedback is used. Non-differential feedback occurs every N TTIs in which the parameters for full precoding matrix are fed back and differential feedback is used for the TTIs between the resets or between non-differential feedbacks in which only the parameters for delta precoding matrix are fed back. Periodic reset time interval N is a design parameter which depends on the channel conditions as well as vehicle speed.

Two different codebooks are used for quantizing differential and non-differential feedback. A codebook consisting of eight codewords is used for non-differential feedback which requires three feedback bits for quantization, while a codebook consisting of four codewords is used for differential feedback which requires only two feedback bits. The codebook used for differential feedback concentrates on the origin of the (θ, ϕ) plane while the codebook for non-differential feedback is uniform with codewords evenly distributed in the predefined vector space. The codebooks are generated using vector quantization and the Lloyd algorithm [10].

The parameters for Jacobi rotation are computed in the diagonalization processing of channel correlation matrix R. The “best” codeword in the codebook that corresponds to the computed parameters for Jacobi rotation is searched. The criterion for searching such codeword is based on the maximization of the norm of the inner product between the computed parameters and the searched codewords. Denote the codebook as C and codewords as c_i and denote the parameters (θ, ϕ) for precoding matrix as w. The “best” codeword for w is the codeword that minimizes the distance in vector space between w and codeword c_i in codebook C,

$$c = \arg\min_{c_i \in C} \sqrt{1 - |c_i^H w|} \qquad (13)$$

This is equivalent to finding the codeword that maximizes the norm of inner product of w and c_i,

$$c = \arg\max_{c_i \in C} |c_i^H w| \qquad (14)$$

Once the codeword is found, the index to the codeword is represented by feedback bits and fed back to transmitter.

3. Numerical Results and Discussions

In this section we present simulation results for the Jacobian feedback scheme for MIMO precoding. The simulation assumption and parameters used are given in Table 1. The MIMO transmission is applied to a SC-FDMA uplink to be used in 3GPP Long Term Evolution [13].

Figure 1 shows the performance of MIMO precoding for a TU6 channel model [13] and vehicle speed at 3km/hr. The performance of MIMO precoding with sub-carrier group feedback of different group sizes is compared. The approach of no group feedback uses the feedback per subcarrier which requires the highest feedback overhead. The approach of sub-carrier group feedback uses one feedback for every L subcarriers. About 0.3 dB degradation is observed for sub-carrier group feedback using one feedback per 12 subcarriers with respect to the performance of no group feedback, i.e., L=1. About 0.8 dB degradation in performance is observed for group feedback using one feedback per 25 subcarriers with respect to no group feedback.

In addition the performance of MIMO precoding with and without quantization is compared. With differential feedback that uses 2 bits per feedback group, about 0.3 dB degradation is observed from quantization for all group feedback sizes, L=1, 12 and 25 subcarriers. The feedback was updated every TTI and was reset every 10 TTIs.

Figure 2 shows the performance comparison for MIMO precoding using differential and non-differential feedback for an SCME-C channel [13]. The performance of combined differential and non-differential feedback that uses a mixed 2 bit/3bit scheme is compared against non-differential feedback using 3 bits. Combined differential and non-differential feedback approach uses 2-bit quantization with 3-bit quantization at each resetting period. The combined differential and non-differential feedback can reduce the feedback overhead by as much as 33% as compared to feedback overhead of non-differential feedback, depending on the iteration interval and reset period. It is observed that the performance of differential feedback using fewer bits (2 bits) with an appropriate resetting interval for differential processing is similar to the performance of non-differential feedback using full feedback and more bits (3 bits). Overall about 0.3-0.4 dB degradation in performance results from using quantization with respect to ideal precoding/TxBF with no quantization.

Figure 3 shows the performance of MIMO precoding using differential feedback with resetting. It is shown that the performance of differential feedback every TTI with proper resetting may improve the performance by 2 dB. This is because the precoding error due to quantization may accumulate or propagate for differential feedback. The resetting process corrects the error, thus improving the performance.

The performance of differential feedback with different resetting intervals of N=10, 20, 30 and 50 TTIs are compared. Performance degradation is negligible; about 0.1 dB degradation in performance is observed with the longest resetting interval of 50 TTIs. Note that this does not account for the effects of possible feedback bit errors; however, we believe that such errors will be rare because of error protection. Also, since the errors will be detectable the same precoding matrix is repeated rather than using an erroneous one.

Figure 4 shows the performance of MIMO precoding using differential feedback with feedback delay for an SCME-C channel and vehicle speed 3 km/h. The combined performance degradation for 2-bit quantization and feedback delay is about 0.3 dB for feedback delay of 2 TTIs and about 0.4 dB for feedback delay of 6 TTIs with respect to no quantization and no feedback delay.

Figure 5 shows the performance of MIMO precoding using differential feedback and feedback delay for an SCME-C channel and vehicle speed 30 km/h. It is shown that about 0.2 dB degradation is observed for feedback delay of 2 TTIs and 1 dB degradation for feedback delay of 6 TTIs with respect to the performance of no feedback delay. The combined performance degradation for 2-bit quantization and feedback delay is about 1.2 dB and 2 dB for feedback delay of 2 and 6 TTIs, correspondingly, with respect to ideal precoding with no quantization and no feedback delay.

Figure 6 shows the performance of MIMO precoding using differential feedback with feedback delay for an SCME-C channel and vehicle speed 120 km/h. It is shown that about 0.6 dB degradation results from 2 TTI feedback delay and about 1.5 dB degradation results from 6 TTI feedback delay with respect to the performance of no feedback delay. When compared to the performance of ideal precoding with no quantization and no feedback, the performance of differential feedback has about 1.7 dB and 2.7 dB degradation for combined quantization and feedback delay of 2 TTIs and 6 TTIs respectively.

Figure 7 shows the performance of MIMO precoding using non-differential feedback for an SCME-C channel and 120 km/h. It is shown that the performance degrades about 0.5 dB for 2 TTI feedback delay and about 2 dB for 6 TTI feedback delay as compared to the performance of no feedback delay. When compared with the performance of ideal precoding with no quantization and no feedback, the performance of differential feedback has about 0.7 dB and 2.2 dB degradation for combined quantization and feedback delay of 2 TTIs and 6 TTIs correspondingly. A shorter feedback delay is obviously preferable for such high speed channels to reduce the performance loss due to speed.

4. Conclusions

In this paper an efficient feedback scheme based on iterative Jacobi rotations is proposed for a MIMO system using either SC-FDMA or OFDMA. Combined differential and non-differential feedback with periodic resetting has been described. It is shown that the differential feedback with proper resetting improves performance. Differential feedback requires considerably less, about 33%, feedback overhead than non-differential feedback while the performance is maintained.

The performance degradation for MIMO precoding due to quantization, sub-carrier group feedback and feedback delay is studied. It is shown that the performance degradation due to quantization for MIMO precoding is within a fractional dB. The performance degradation of MIMO precoding due to group feedback depends on the channel coherent bandwidth and the size of the feedback group. The loss is within 1 dB for feedback every 25 subcarriers. It is also shown that performance degradation due to feedback delay is within a fractional dB for low speed or shorter feedback delay such as 3 km/h or feedback delay of 2 TTIs. The performance continues to degrade as either the vehicle speed or amount of feedback delay increases.

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Table 1 Simulation Parameters

Parameter	Assumption
Carrier frequency	2.0 GHz
Sampling frequency	7.68 MHz
Transmission bandwidth	5 MHz
TTI length	0.5 ms
Number of long/short blocks per TTI	6/2
Number of occupied subcarriers	300
FFT block size	512
Number of used subcarriers for data	256
Cyclic Prefix (CP) length	5.078 us (39 samples)
Channel model	Typical Urban (TU6), SCME-C
Antenna configurations	2 x 2 (MIMO)
Fading correlation between transmit/receive antennas	ρ = 0 for TU6, and SCME-C
Moving speed	3 km/hr, 30 km/hr, 120 km/hr
Data modulation	QPSK and 16QAM
Channel coding	Turbo code with soft-decision decoding
Coding rate	½ and 1/3
Equalizer	LMMSE
Group feedback	One feedback per 1, 12 and 25 subcarriers
Feedback error	None (Assumed ideal)
Feedback delay	2 and 6 TTIs
Channel Estimation	Ideal channel estimation

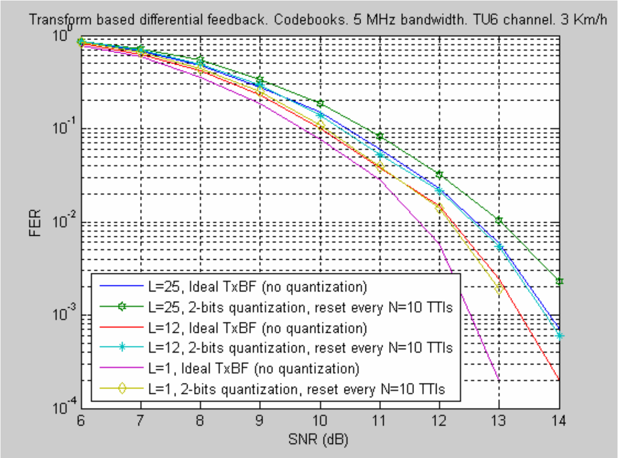

Figure 1: Performance for MIMO precoding using group feedback and codebook quantization for TU6 channel and vehicle speed 3 km/h.

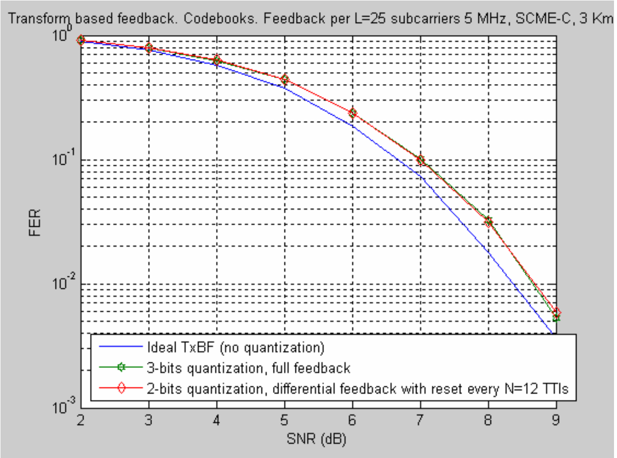

Figure 2: Performance comparison for MIMO precoding using differential and non-differential feedback.

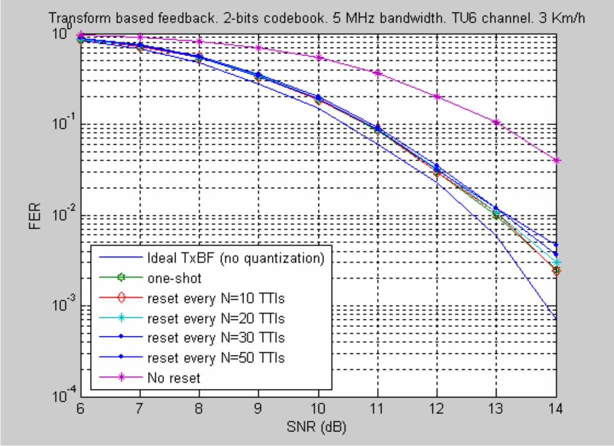

Figure 3: Performance of MIMO precoding using differential feedback with resetting.

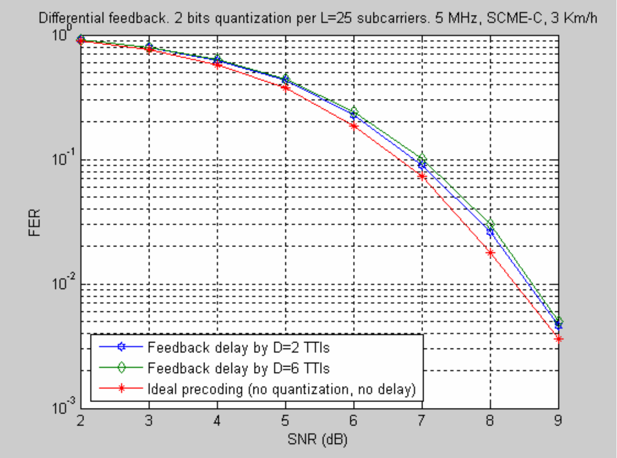

Figure 4: Performance of MIMO precoding using differential feedback with feedback delay for SCME-C, 3km/h.

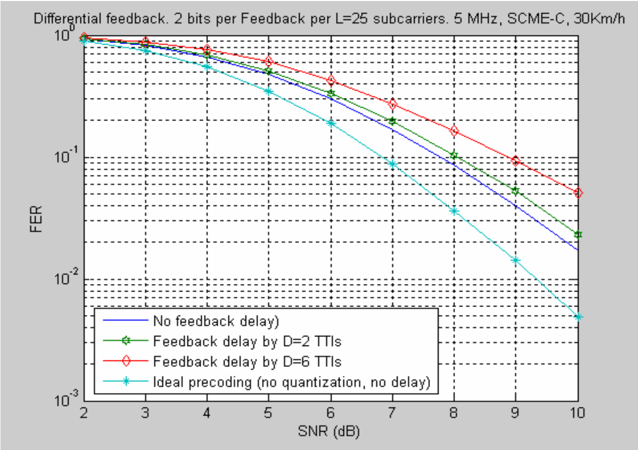

Figure 5: Performance of MIMO precoding using differential feedback and feedback delay for SCME-C, 30km/h.

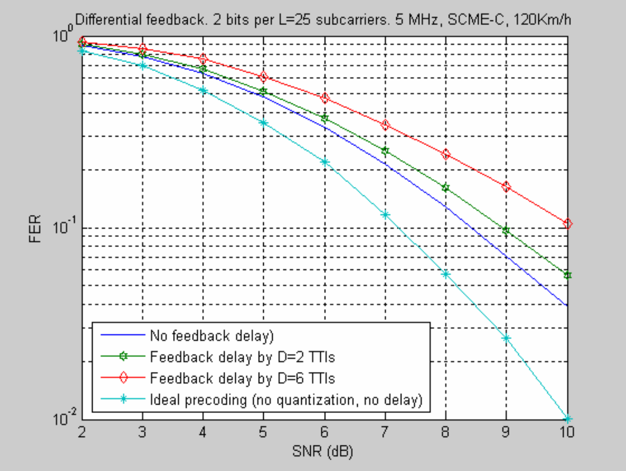

Figure 6: Performance of MIMO precoding using differential feedback and feedback delay for SCME-C, 120km/h.

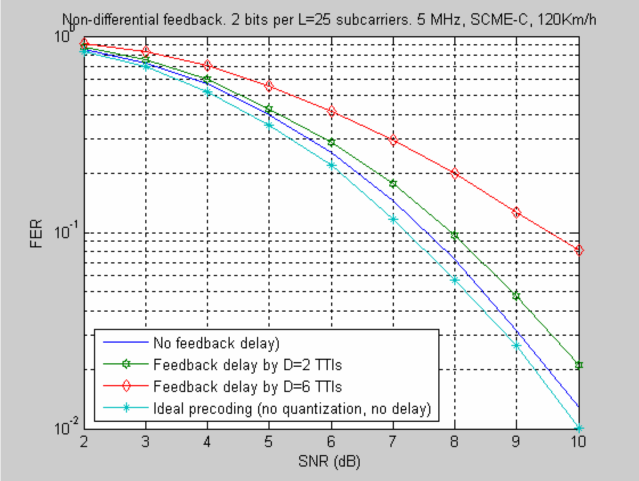

Figure 7: Performance of MIMO precoding using non-differential feedback and feedback delay for SCME-C, 120km/h.