Peak Power Characteristics of Single Carrier FDMA MIMO Precoding System

Hyung G. Myung*, Jung-Lin Pan†, Robert Olesen†, and Donald Grieco†

* Dept. of Electrical and Computer Engineering
Polytechnic University
Brooklyn, NY 11201, USA

† InterDigital Communications Corp.
2 Huntington Quadrangle
Melville, NY 11747, USA

***Abstract*— A salient advantage of single carrier frequency division multiple access (SC-FDMA) is low peak-to-average power ratio (PAPR) which is lower than that of OFDMA. Precoding using transmit beamforming (TxBF) is a MIMO spatial multiplexing method that increases the data throughput significantly. However, when applied to SC-FDMA, it increases the PAPR. In this paper, we describe how we can apply TxBF to an SC-FDMA system and show its effect on PAPR. We then consider the effects of precoder averaging across subcarriers and quantization, typically used to reduce feedback overhead, on the PAPR. Comparisons are made with single antenna, non-precoded spatial multiplexing (SM) and space-frequency block coding (SFBC) systems. Lastly, we show that a modest amount of amplitude clipping can reduce PAPR to an acceptable level with virtually no harmful effects on performance or spectral growth.**

***Index Terms*—peak-to-average power ratio, single carrier FDMA, MIMO, precoding, clipping**

I. INTRODUCTION

Broadband wireless communications suffer from multi-path frequency-selective fading which significantly degrades the system performance. Orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA) are multicarrier communication techniques which are robust against frequency selective fading channels. But they suffer from high peak-to-average power ratio (PAPR) which degrades the transmit power efficiency.

Single carrier frequency division multiple access (SC-FDMA), which utilizes single carrier modulation and frequency domain equalization, is a technique that has similar performance and essentially the same overall structure as those of OFDMA system [1]. One prominent advantage over OFDMA is that the SC-FDMA signal has lower PAPR because of its inherent single carrier structure [2]. SC-FDMA has drawn great attention as an attractive alternative to OFDMA, especially in the uplink communications where lower PAPR greatly benefits the mobile terminal in terms of transmit power efficiency. SC-FDMA has been adopted for the uplink multiple access scheme for the 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE), or Evolved UTRA [3], [4].

Multiple input multiple output (MIMO) spatial multiplexing technique offers an attractive way to significantly increase the peak data rate. Among the various MIMO techniques, precoding or transmit eigen-beamforming (TxBF) is a promising technique with many benefits [5], [6], [7]. However, the use of precoding such as TxBF may increase the PAPR because each transmit signal becomes a composite signal due to spatial processing.

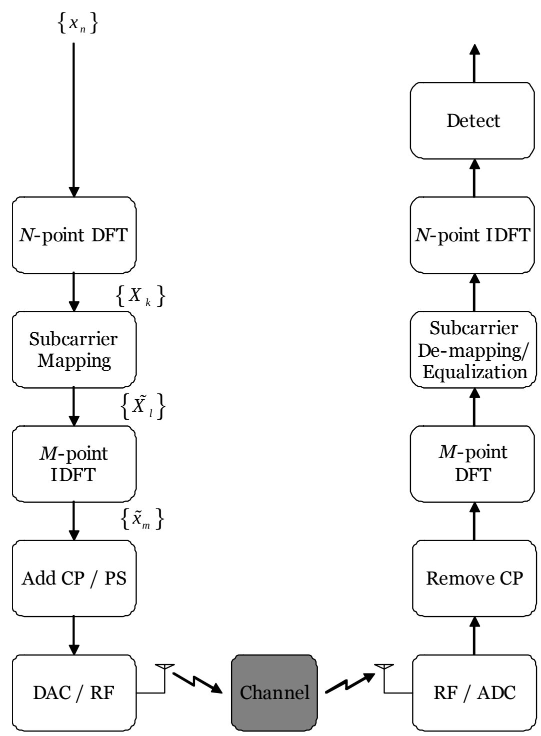

Figure 1: Transmitter and receiver structure of an SC-FDMA system.

In this paper, we first describe how TxBF can be applied to an SC-FDMA system and show its effect on PAPR. We then consider the effects of precoder averaging across subcarriers and quantization, typically used to reduce feedback overhead, on the PAPR. Comparisons are made with single antenna, non-precoded spatial multiplexing (SM) and space-frequency block coding (SFBC) systems. We also describe a method to reduce PAPR using symbol amplitude clipping.

II. SINGLE CARRIER FDMA (SC-FDMA)

Figure 1 illustrates the transmitter and receiver structure of SC-FDMA. At the input to the transmitter, a baseband modulator transforms the binary input to a multilevel sequence of complex numbers in one of several possible modulation formats. The system adapts the modulation format, and thereby the transmission bit rate, to match the current channel conditions of each terminal.

The transmitter next groups the modulation symbols, x_n into blocks each containing N symbols. The first step in modulating the SC-FDMA subcarriers is to perform an N-point discrete Fourier transform (DFT), to produce a frequency domain representation X_k of the input symbols. It then maps each of the N DFT outputs to one of the M (> N) orthogonal subcarriers that can be transmitted. Typically, $N = M/Q$ is an integer submultiple of M. Q is the bandwidth expansion factor of the symbol sequence. For example, if all terminals transmit N symbols per block, the system can handle Q simultaneous transmissions without co-channel interference. The result of the subcarrier mapping is the set of complex subcarrier amplitudes, where N of the amplitudes are non-zero. As in OFDMA, an M-point inverse DFT (IDFT) transforms the subcarrier amplitudes to a complex time domain signal. Each then modulates a single frequency carrier and all the modulated symbols are transmitted sequentially.

The transmitter performs two other signal processing operations prior to transmission. It inserts a set of symbols referred to as a cyclic prefix (CP) in order to provide a guard time to prevent inter-block interference (IBI) due to multipath propagation. The transmitter also performs a linear filtering operation referred to as pulse shaping in order to reduce out-of-band signal energy.

The receiver transforms the received signal into the frequency domain via DFT, de-maps the subcarriers, and then performs frequency domain equalization. Because SC-FDMA uses single carrier modulation, it suffers from inter-symbol interference (ISI) and thus equalization is necessary to combat the ISI [8]. The equalized symbols are transformed back to the time domain via IDFT, and detection and decoding take place in the time domain.

Disadvantages of OFDMA compared to SC-FDMA are its strong sensitivity to carrier frequency offset and strong sensitivity to nonlinear distortion in the power amplifier due to the high PAPR, both properties of the multicarrier nature of OFDMA. PAPR was a major factor in selecting SC-FDMA over OFDMA as the uplink air interface for 3GPP LTE.

III. SC-FDMA MIMO PRECODING/TRANSMIT EIGEN-BEAMFORMING SYSTEM

We can use spatial multiplexing MIMO techniques in SC-FDMA by applying the spatial processing on a subcarrier-by-subcarrier basis, somewhat similar to MIMO-OFDM [9], in the frequency domain after DFT. A block diagram of a generic spatial multiplexing MIMO SC-FDMA system is shown in Figure 2. Transmit eigen-beamforming

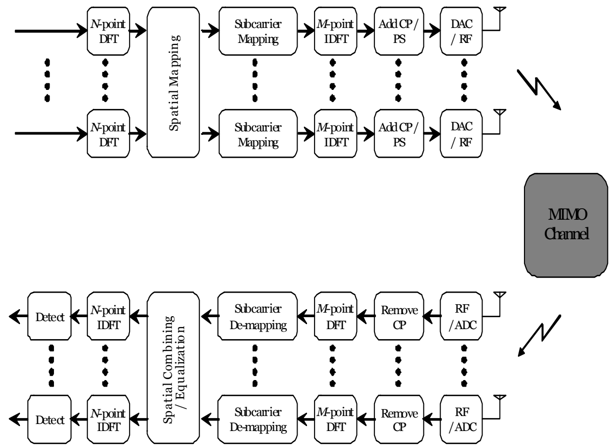

Figure 2: Block diagram of a spatial multiplexing SC-FDMA MIMO system.

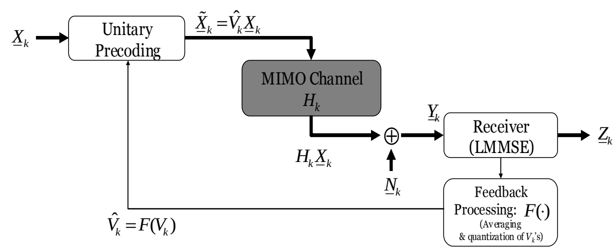

Figure 3: Simplified block diagram of unitary precoded TxBF SC-FDMA MIMO system.

(TxBF) with unitary precoding utilizes the eigen-structure of the channel to achieve spatial multiplexing. Figure 3 illustrates a simplified block diagram of a unitary precoded TxBF SC-FDMA MIMO system. We describe below the basic processes of unitary precoded TxBF system. We assume the channel is time-invariant and the channel estimation is perfect.

First, the MIMO channel matrix H_k for subcarrier k is decomposed as follows using singular value decomposition (SVD) for each subcarrier at the receiver.

$$H_k = U_k D_k V_k^H \tag{1}$$

We feedback V_k to the transmitter and use it as the precoding matrix for transmission. We may quantize and average the V_k's at the receiver to reduce feedback overhead. We denote the quantization and averaging processes as $F(\cdot)$ and $\hat{V}_k = F\left(V_k\right)$. The quantized and averaged $\hat{V}_k$'s are then sent to the transmitter via the feedback channel. The transmitter precodes the data signal $\underline{X}_k$ with $\hat{V}_k$ as follows.

$$\underline{\tilde{X}}_k = \hat{V}_k \underline{X}_k \tag{2}$$

After the transmitted signal propagates through the MIMO channel H_k, the received signal $\underline{Y}_k$ is represented as follows.

$$\underline{Y}_k = H_k \underline{\tilde{X}}_k + \underline{N}_k \tag{3}$$

At the receiver, we perform linear minimum mean square

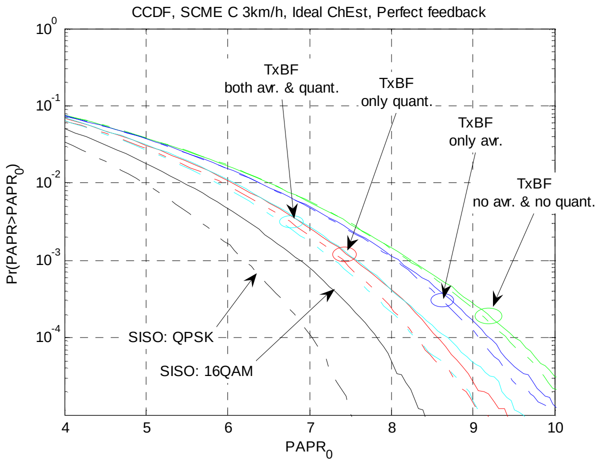

Figure 4: PAPR of 2x2 SC-FDMA MIMO TxBF.

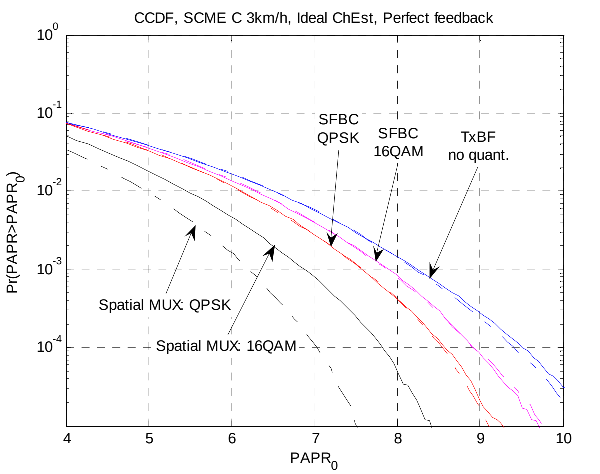

Figure 5: PAPR of different 2x2 SC-FDMA MIMO schemes.

error (LMMSE) estimation for joint equalization and MIMO detection. Let $\underline{Z}_k = A_k \underline{Y}_k$ where A_k is an $N_t \times N_r$ complex matrix where N_t is the number of transmit antenna and N_r is the number of receive antenna. Then, our goal is to find A_k such that mean square error between $\underline{X}_k$ and $\underline{Z}_k$ ($E\left\{\left|\underline{X}_k - \underline{Z}_k\right|^2\right\}$) is minimum. By applying orthogonality principal, we obtain the following solution for A_k.

$$A_k = R_{XX,k}\hat{H}_k^H \left(\hat{H}_k R_{XX,k} \hat{H}_k^H + R_{NN,k}\right)^{-1} \tag{4}$$

where $\hat{H}_k = H_k \hat{V}_k$, $R_{XX,k} = \underline{X}_k \underline{X}_k^H$, and $R_{NN,k} = \underline{N}_k \underline{N}_k^H$.

IV. PAPR Properties of an SC-FDMA MIMO Precoding TxBF System

Precoding in the frequency domain is convolution (filtering) and summation in the time domain. Thus we expect to see an increase in the peak power because of the filtering and summation.

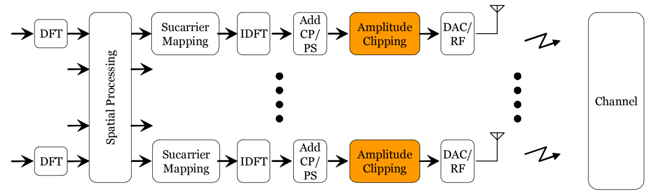

Figure 6: Symbol amplitude clipping for SC-FDMA MIMO transmission.

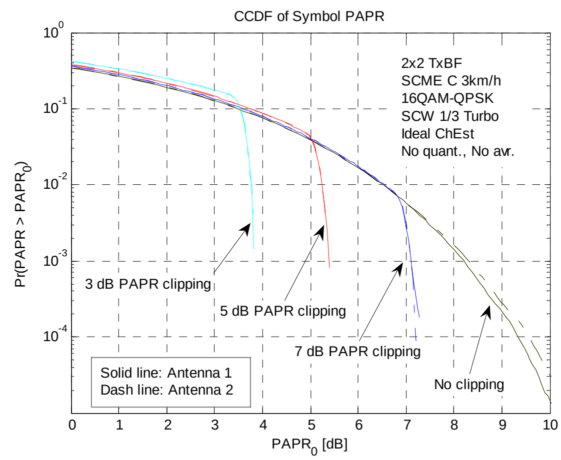

Figure 7: CCDF of symbol PAPR.

Unlike OFDM, statistical properties of PAPR for single carrier modulations are not easily obtained analytically [10]. We thus resort to numerical analysis to investigate the PAPR properties of our SC-FDMA MIMO system.

Figure 4 illustrates the PAPR characteristics of a 2x2 SC-FDMA MIMO TxBF signal with 16QAM and QPSK in each stream. Simulation parameters for subsequent simulations are shown in Table 1. The channel is averaged over 25 continuous subcarriers. Direct quantization of the precoder matrix using 3 bits (1 bit for amplitude and 2 bits for phase information) was performed.

Without averaging of channel and quantization of precoding matrix, the MIMO TxBF signal has 1.5 ~ 2 dB higher PAPR with respect to single antenna transmission. When averaging of the channel estimate over a resource block (RB) of 25 subcarriers and quantization are considered, the PAPR for MIMO TxBF decreases by 0.8 ~ 0.9 dB and has about 0.5 dB higher PAPR for QAM with respect to single antenna transmission. This results from low-pass filtering caused by block averaging and clipping caused by amplitude quantization.

Figure 5 compares the PAPR of different MIMO schemes. Since there is no precoding or spatial processing at the transmitter for non-precoded spatial multiplexing (SM), SM has the same PAPR as single antenna transmission. Without quantization for precoding matrix, TxBF has higher PAPR than SFBC by 0.5 ~ 1 dB. However, TxBF and SFBC have similar PAPR when quantization of precoding matrix is considered.

V. PAPR Reduction by Symbol Amplitude Clipping Method in SC-FDMA

A. Symbol Amplitude Clipping

There are many techniques developed to reduce PAPR for multi-carrier transmission that may be applicable to SC-FDMA [11]. One common way to reduce PAPR is to limit or clip the peak power of the transmitted symbols. The problems associated with clipping are in-band signal distortion and generation of out-of-band signal. Because SC-FDMA modulation spreads the information data across all the modulated symbols, in-band signal distortion is mitigated when an SC-FDMA symbol is clipped. Figure 6 shows the block diagram of a simple symbol amplitude clipping method for SC-FDMA MIMO transmission. Other sophisticated methods, such as multi-stage clipping or soft clipping, can also be considered.

In the subsequent section, we show simulation results for clipping applied to 2x2 uplink SC-FDMA MIMO precoding TxBF scheme.

B. Symbol Amplitude Clipping for 2x2 SC-FDMA MIMO TxBF

Figure 7 shows the CCDF of symbol PAPR with clipping at various levels. With 7 dB PAPR clipping, less than 1% of the symbols are clipped. Note that even with as much as 3 dB PAPR clipping, only about 10% of the modulated symbols are clipped.

Figures 8 and 9 show the link level performance when clipping is applied. It is observed that the performance degradation due to clipping is minimal when clipping at 7 dB above the average power. The performance degrades slightly more when clipping is increased to a level 5 or 3 dB above the average power.

Clipping, consequently, will generate both in-band and out-of-band frequency components. Figure 10 shows the frequency spectrum of the clipped signals by power spectral density (PSD). For PSD calculation, a Hanning window was used with 1/4 of window overlapping.

For 7 dB PAPR clipping, the spectrum is almost the same as that of the original signal. More pronounced out-of-band components arise when 5 or 3 dB PAPR clipping is used. It is clear that 7 dB clipping has minimal impact on the power spectrum.

VI. Conclusions

In this paper, we investigated the PAPR properties for an SC-FDMA uplink system using 2x2 MIMO with unitary precoded TxBF. We have shown that precoding such as unitary precoded TxBF or SFBC increases the PAPR for an SC-FDMA signal. We also found out that averaging and quantization of precoder actually reduce the PAPR. As a result, the PAPR with beamforming is comparable to that of SFBC and is about 0.5 dB higher than the single antenna or non-precoded SM MIMO transmission. A modest amount of amplitude clipping, e.g. 7 dB above the average power level, can be used to further reduce the PAPR for beamforming with SC-FDMA without compromising

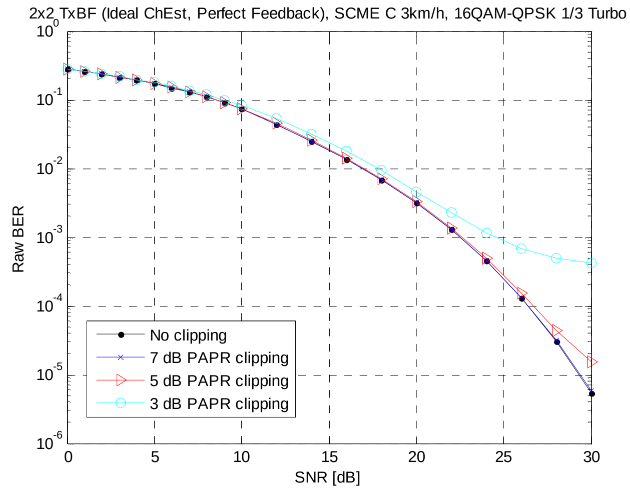

Figure 8: Raw BER performance of clipping.

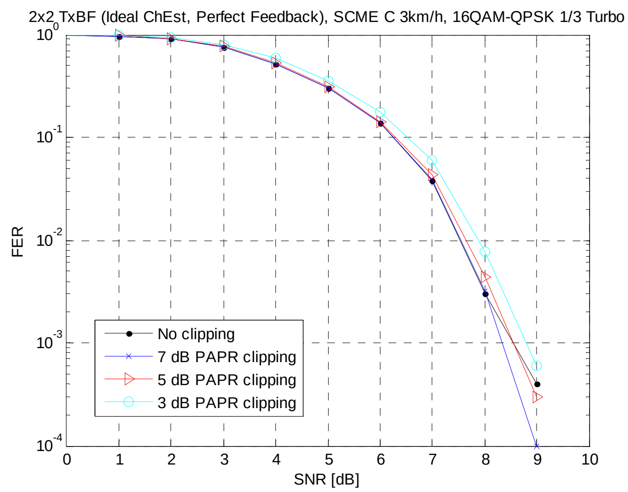

Figure 9: FER performance of clipping.

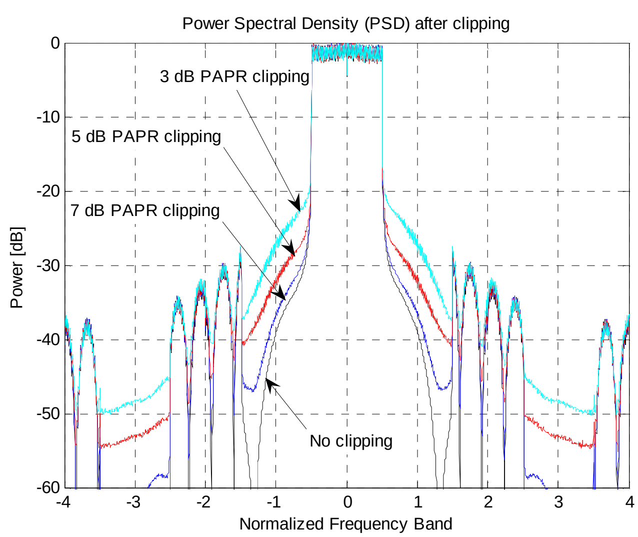

Figure 10: PSD after clipping.

the link level performance or increasing out-of-band emissions. Therefore, one can obtain the benefit of improved performance provided by TxBF or other precoder MIMO schemes while maintaining the desirable low PAPR property of SC-FDMA.

REFERENCES

[1] H. G. Myung, J. Lim, and D. J. Goodman, “Single Carrier FDMA for Uplink Wireless Transmission”, *IEEE Vehicular Technology Mag*., vol. 1, no. 3, Sep. 2006, pp. 30-38.

[2] H. G. Myung, J. Lim, and D. J. Goodman, "Peak-to-average Power Ratio of Single Carrier FDMA Signals with Pulse Shaping", *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)* 2006, Sep. 2006.

[3] H. Ekström, A. Furuskär, J. Karlsson, M. Meyer, S. Parkvall, J. Torsner, and M. Wahlqvist, “Technical Solutions for the 3G Long-Term Evolution,” *IEEE Commun. Mag*., vol. 44, no. 3, Mar. 2006, pp. 38-45.

[4] 3GPP TR 25.814 – 3GPP; Technical Specification Group Radio Access Network; Physical Layer Aspects for Evolved Universal Terrestrial Radio Access (UTRA) (Release 7).

[5] J.-L. Pan, R. L. Olesen, D. Grieco, and C.-P. Yen, “Efficient Feedback Design for MIMO SC-FDMA Systems,” *to appear in IEEE VTC ’07 Spring*, Dublin, Irelend, Apr. 2007.

[6] InterDigital, “Extension of Uplink MIMO SC-FDMA with Preliminary Simulation Results,” *3GPP TSG-RAN WG1 #44*, Technical Document R1-060365, Feb. 2006.

[7] J. S. Hammerschmidt, C. Brunner, and C. Drewes, “Eigenbeamforming - A Novel Concept in Array Signal Processing,” *Proc. European Wireless Conference 2000*, Dresden, Germany, Sep. 2000.

[8] D. Falconer, S. L. Ariyarisitakul, A. Benyamin-Seeyar, and B. Eidson, “Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems,” *IEEE Commun. Mag.*, vol. 40, no. 4, Apr. 2002, pp. 58-66.

[9] G. L. Stüber, J. R. Barry, S. W. McLaughlin, Y. G. Li, M. A. Ingram, and T. G. Pratt, “Broadband MIMO-OFDM Wireless Communications,” *Proceedings of the IEEE*, vol. 92, no. 2, Feb. 2004, pp. 271-294.

[10] D. Wulich and L. Goldfeld, “Bound of the Distribution of Instantaneous Power in Single Carrier Modulation,” *IEEE Trans. Wireless Commun*., vol. 4, no. 4, Jul. 2005, pp. 1773–1778.

[11] S. H. Han, and J. H. Lee, “An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission,” *IEEE Wireless Commun.*, vol. 12, no. 2, Apr. 2005, pp. 56-65.

[12] Ericsson, Elektrobit, Nokia, Motorola, and Siemens, “LTE Channel Models and Simulations,” *3GPP TSG-RAN WG4 #38*, Technical Document R4-060334, Feb. 2006.

TABLE 1. SIMULATION PARAMETERS.

Parameter	Assumption
Carrier frequency	2.0 GHz
Symbol rate	7.68 million symbols/sec
Transmission bandwidth	5 MHz
TTI length	0.5 ms
Number of data blocks per TTI	6 long blocks (LB)
Number of occupied subcarriers per LB	128
FFT block size	512
Cyclic Prefix (CP) length	32 samples
Subcarrier mapping	Distributed
Pulse shaping	Time domain RRC filter (rolloff = 0.22)
Oversampling	2x oversampling
Channel model	3GPP SCME C, 3 km/h [12]
Antenna configurations	2 x 2 (MIMO)
Data modulation	QPSK and 16QAM
Channel coding	Turbo code with R = 1/3
Equalizer	LMMSE
Feedback error	None
Channel Estimation	Perfect channel estimation